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Strings on type-IIB pp-wave backgrounds with interacting massive theories on the worldsheet

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Abstract: We consider superstring theories on pp-wave backgrounds which result in an integrable $\mathcal{N} = (2, 2)$ supersymmetric Landau-Ginzburg theory on the worldsheet. We obtain exact eigenvalues of the light-cone gauge superstring hamiltonian in the massive and interacting world-sheet theory with superpotential $Z^3 - Z$. We find the modes of the supergravity part of the string spectrum, and their space-time interpretation. Because the system is effectively at strong coupling on the worldsheet, these modes are not in one-to-one correspondence with the usual type-IIB supergravity modes in the $p_- \to 0$ limit. However, the above correspondence holds in the $\alpha' \to 0$ limit.

Keywords: Superstrings and Heterotic Strings, Penrose limit and pp-wave background, Integrable Field Theories.

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1. Introduction

Since the discovery of the AdS/CFT correspondence [1], superstring theories on Ramond-Ramond backgrounds have been studied extensively. These theories are also interesting because the RNS formalism cannot be applied; one needs to use the Green-Schwarz formalism. The Penrose limit of the $AdS_5 \times S_5$ background gives a pp-wave background which has been studied in the context of AdS/CFT correspondence [2]. These maximally supersymmetric pp-wave solutions of IIB supergravity were considered in [3]. The maximally supersymmetric pp-wave background supported by a constant R-R five-form field strength is also a useful framework, since the string theory on it can be solved exactly [4, 5]. Since D-branes are expected to be dual to non-perturbative objects in gauge theory, they play an important role in AdS/CFT correspondence. D-branes on maximally supersymmetric pp-wave backgrounds have been studied in [6]–[9].

One remarkable feature of the theories studied in [4, 5] is that in the light-cone gauge GS formalism, they result in massive theories on the world sheet. This means that one cannot use conformal field theory when studying the physics from the two-dimensional point-of-view. The world-sheet theories of [4, 5] are free, however, so the spectrum of the light-cone gauge hamiltonian can still be easily found. However, this is not true for another interesting class of pp-wave type backgrounds which are solutions of type-IIB supergravity with non-constant R-R five-form field strength [10]. In the case of flat transverse space these backgrounds provide exact solutions of superstring theory [11]. String theories on these backgrounds give interacting $\mathcal{N} = (2,2)$ or $\mathcal{N} = (1,1)$ supersymmetric field theories on the worldsheet. The first class is parametrized in terms of an arbitrary holomorphic function which becomes the worldsheet superpotential in the light-cone gauge superstring action. Generic superpotentials are massive, so conformal field theory cannot be applied. One can obtain exact information about the spectrum in generic $\mathcal{N} = (2,2)$ theories [12], but except in simple cases it is difficult to extract quantitative information from this formalism.

It is therefore useful to study pp-wave backgrounds which lead to worldsheet superpotentials describing integrable massive theories. In these theories, several techniques make possible the computation of the spectrum as a function of worldsheet mass and size. The technique we will utilize here is a generalization of the thermodynamic Bethe ansatz [13] to compute excited-state energies [14]. In fact, the finite-size spectrum for the simplest $\mathcal{N} = (2,2)$ massive field theory, that with superpotential $Z^3 - Z$, has already been derived [15].

In this paper, we discuss how to utilize these results in string theory on type-IIB supergravity backgrounds. The transverse target space is taken flat and non-compact. Two of the eight transverse space directions are interacting while the others are free. On non-compact two dimensional space, the 2d quantum field theory with the above superpotential has the spectrum consisting of kinks and antikinks interpolating between the two supersymmetric vacua. When considered on a cylinder, as necessary for closed strings, the states consist of pairs of interacting kinks and antikinks [15]. In target space they correspond to strings interpolating between two planes sitting at the minima of the superpotential.

The supergravity part of the string spectrum can be obtained by considering all states contributing to the string's point particle limit, which can be obtained by taking either the $\alpha' \to 0$ or $p_- \to 0$ limit. In the first case we obtain the type-IIB supergravity spectrum as the worldsheet theory reduces to a free massive theory in the limit. In the second case, strings with $p_- \to 0$ explore the UV limit of the worldsheet theory, which gives the IR limit of the spacetime theory. In this limit we expect to obtain the IIB supergravity spectrum but, as we will see, we do not obtain all of it. In the limit $p_- \to 0$ the supergravity part of the string spectrum comes from states with no kinks or antikinks, and no oscillation modes from the free part of the light-cone gauge string lagrangian. However, since there are no fermionic zero modes corresponding to the interaction part of the light-cone gauge string lagrangian, the Clifford algebra satisfied by the GS fermionic zero modes is degenerate. This is different from the usual non-degenerate Clifford algebra in the case of flat, usual pp-wave string theories or our case when the limit $\alpha' \to 0$ is considered. In the case where the superpotential is for only one of the four complex transverse coordinates of the string, the supergravity part of the string spectrum consists of 128 instead of 256 states corresponding to IIB supergravity. This background has an SO(3) global symmetry. Under this symmetry we find the space-time fields associated to the supergravity part of the string spectrum. Some IIB supergravity modes are absent when $p_- \to 0$, essentially because the world-sheet system is at strong coupling, far from the free-particle limit of [4, 5].

There has been a fair amount of related work on pp-wave type backgrounds. Such backgrounds with non-trivial R-R fluxes or other fields, which can still be solved exactly at least in some cases, have been analyzed recently in [16]–[19]. In general these backgrounds preserve 16 supersymmetries and the light-cone gauge string lagrangian contains harmonic oscillatory type terms in the fields. The question of the amount of surviving supersymmetry in D-brane configurations on pp-wave type backgrounds found in [10] has been addressed in [20]. A related background with localized D-branes solutions on general pp-wave backgrounds has been found in [21]. Other backgrounds producing interacting light-cone gauge string models have been considered in the past [22, 23] and more recently in [24]–[27].

In section 2 we briefly review the general pp-wave solutions of type-IIB supergravity found in [10]. In section 3 we find the eigenvalues of the light-cone gauge string hamiltonian in the background corresponding to the superpotential $Z^3 - Z$. The supergravity part of superstring spectrum as well as its space-time interpretation is analyzed in section 4. In section 5 we discuss other cases when there are 2,3 or 4 interacting theories. Also, we mention the background leading to $\mathcal{N} = 2$ supersymmetric sine-Gordon theory on the worldsheet. Finally, we present our conclusions in section 6.

2. Superstrings on general IIB pp-wave backgrounds

Maximally-supersymmetric pp-wave solutions of IIB supergravity were considered in [3]. Strikingly, the world-sheet theory is massive, but since the fields are free, the eigenvalues of the light-cone superstring hamiltonian can still be found [4, 5]. The type-IIB supergravity equations also admit more general pp-wave type solutions of the form:

$$\begin{aligned} ds^2 &= -2dx^+dx^- + H(x^i)dx^+dx^+ + g_{ij}dx^idx^j \\ F_5 &= dx^+ \wedge \varphi_4(x^i)\,, \end{aligned} \tag{2.1}$$

where $x^i, i = 1, \ldots, 8$ are the 8 transverse coordinates, $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^9)$ and φ_4 a four form in the transverse space. The transverse space can be curved. All other fields are zero except the dilaton, which is constant. The equations of motion of type-IIB supergravity imply that the transverse space is Ricci flat, the five-form F_5 is self-dual and that the four-form φ_4 and function H are related by:

$$\nabla^2 H = -32\,|\varphi|^2\;, \qquad *_{10}F_5 = F_5\,, \qquad R_{ij}[g] = 0 \tag{2.2}$$

where $|\varphi|^2 = \frac{1}{4!}\varphi_{\mu\nu\lambda\delta}\varphi^{\mu\nu\lambda\delta}$, and ∇^2 is the laplacian in the transverse space. Since there are no other forms F_5 is closed which along with the self-duality of F_5 implies $*\varphi = -\varphi$ and $d\varphi = 0$. In complex coordinates the general 4-forms can be classified by the (a, b), the number of holomorphic and antiholomorphic indices, with $a + b = 4$. The anti-self dual 4-forms are either $(1, 3)$, their complex conjugates $(3, 1)$, or $(2, 2)$, all of which can be written explicitly as

$$\varphi_{mn} = \frac{1}{6}\varphi_{m\bar{i}\bar{j}\bar{k}}\epsilon^{\bar{i}\bar{j}\bar{k}\bar{n}}g_{n\bar{n}}\,, \qquad \varphi_{l\bar{m}} = \frac{1}{2}g^{s\bar{s}}\varphi_{l\bar{m}s\bar{s}}, \tag{2.3}$$

where φ_{mn} is a symmetric matrix and $\varphi_{l\bar{m}}$ is a hermitian traceless matrix.

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The amount of supersymmetry preserved by this pp-wave type background was studied in [10]. The supersymmetry transformations are generated by a chiral spinor ϵ with 16 complex components. Under the embedding $\mathrm{SO}(1,1)\times\mathrm{SO}(8)\subset\mathrm{SO}(9,1)$ the spinor ϵ is decomposed in positive and negative SO(1,1) and SO(8) chiralities ($\Gamma's$ are 32×32 matrices):

$$\epsilon=-\frac{1}{2}\Gamma_+\Gamma_-\epsilon-\frac{1}{2}\Gamma_-\Gamma_+\epsilon\equiv\epsilon_++\epsilon_-\,. \tag{2.4}$$

The amount of supersymmetry preserved is obtained by solving the Killing spinor equation. To do this it is convenient to work in complex transverse space $z^i=\frac{1}{\sqrt{2}}(x^i+ix^{i+4})$, with $i=1,2,3,4$. However, later in this paper when we analyze strings on these IIB supergravity backgrounds we switch back to real coordinates.

Two classes of solutions have been found in [10], for both flat and curved transverse space. The first class preserves at least four spacetime supersymmetries and the second at least two. For the flat case, which we consider in this paper, these solutions can be parametrized by a holomorphic function W and a 4×4 hermitian traceless constant matrix $\varphi_{j\bar{k}}$. The Killing spinors are:

$$\begin{aligned}\epsilon_+&=\alpha|0\rangle+\zeta|\tilde{0}\rangle\,,\\ \epsilon_-&=2i\Gamma_-\left(\zeta\partial_{\bar{k}}\overline{W}-\alpha\varphi_{j\bar{k}}z^j\right)\Gamma^{\bar{k}}|0\rangle+2i\Gamma_-\left(\alpha\partial_kW-\zeta\varphi_{k\bar{j}}\bar{z}^{\bar{j}}\right)\Gamma^k|\tilde{0}\rangle\,,\end{aligned} \tag{2.5}$$

where α and ζ are constant complex parameters. In the Fock space formalism the vacuum $|0\rangle$ and the filled state $|\tilde{0}\rangle=\frac{1}{4}\Gamma^{\bar{1}}\Gamma^{\bar{2}}\Gamma^{\bar{3}}\Gamma^{\bar{4}}|0\rangle$ are given by:

$$\Gamma_+|0\rangle=\Gamma^n|0\rangle=0\,,\qquad\Gamma_+|\tilde{0}\rangle=\Gamma^{\bar{n}}|\tilde{0}\rangle=0\,,\qquad n=1,2,3,4\,. \tag{2.6}$$

In terms of W the metric and the non-constant matrices φ_{mn} and $\varphi_{\bar{m}\bar{n}}$ are:

$$\begin{aligned}ds^2&=-2dx^+dx^--32(|\partial_iW|^2+|\varphi_{j\bar{i}}z^j|^2)dx^+dx^++2dz^id\bar{z}^{\bar{i}}\\ \varphi_{mn}&=\partial_m\partial_nW\,,\qquad\varphi_{\bar{m}\bar{n}}=\partial_{\bar{m}}\partial_{\bar{n}}\overline{W}\,.\end{aligned} \tag{2.7}$$

The second class of solutions, preserving at least two supersymmetries, are parametrized by a real harmonic function U. The Killing spinors are given by:

$$\begin{aligned}\epsilon_+&=-\zeta|0\rangle+\zeta|\tilde{0}\rangle\,,\\ \epsilon_-&=2i\Gamma_-\zeta\partial_{\bar{k}}U\Gamma^{\bar{k}}|0\rangle-2i\Gamma_-\zeta\partial_kU\Gamma^k|\tilde{0}\rangle\,.\end{aligned} \tag{2.8}$$

The metric and the matrices φ are:

$$\begin{aligned}ds^2&=-2dx^+dx^--32|\partial_kU|^2dx^+dx^++2dz^id\bar{z}^{\bar{i}}\,,\\ \varphi_{mn}&=\partial_m\partial_nU\,,\qquad\varphi_{\bar{m}\bar{m}}=\partial_{\bar{m}}\partial_{\bar{n}}U\,,\qquad\varphi_{l\bar{m}}=\partial_l\partial_{\bar{m}}U\end{aligned} \tag{2.9}$$

In the rest of this paper, we consider only the first class of the above two sets of solutions. When strings are considered on the above pp-wave type backgrounds, the light-cone gauge can be implemented [28]. We choose the light-cone gauge by setting $X^+=\tau$, and $\Gamma^+\theta=\Gamma^+\tilde{\theta}=0$, where τ is the worldsheet time and $\theta,\tilde{\theta}$ the two Majorana-Weyl spinors. Since $(\Gamma^+)^2 = 0$ they become the usual Green-Schwarz SO(8) spinors $S, \tilde{S}$. In the usual light-cone gauge procedure, p_- is conserved and the light-cone gauge hamiltonian is $H = -p_+$. The remaining killing spinors ϵ_+ are those not annihilated by Γ^+. These spinors survive as linearly-realized supersymmetries on the worldsheet. When all Ramond-Ramond fields are zero, the superstring action becomes an $\mathcal{N} = (2,2)$ supersymmetric non-linear sigma model written in terms of a Kähler potential. When the R-R five-form field strength F_5 is turned on (background (2.7)), one expects to still have a supersymmetric theory on the worldsheet, due to the preserved spinors ϵ_+. As shown in [10], when the forms $(1,3)$ and $(3,1)$ are present one can add an arbitrary superpotential W so that the worldsheet action in terms of the superfields is:

$$\begin{aligned} S &= \frac{1}{4\pi\alpha'}\int d\tau \int_0^{2\pi\alpha'|p_-|} d\sigma (L_K + L_W) \\ L_K + L_W &= \int d^4\theta g_{i\bar{j}} \Phi^i \bar{\Phi}^{\bar{j}} + \frac{1}{2}\left(\int d^2\theta W(\Phi^i) + \text{c.c.}\right), \end{aligned} \tag{2.10}$$

where Φ^i are chiral superfields with $i = 1,2,3,4$. An interesting feature of this action is that it describes an interacting field theory. In complex coordinates we have the subgroup SU(4) of SO(8), and a negative chirality SO(8) spinor decomposes in spinors with vector indexes as $\mathbf{8}_- \to \mathbf{4} + \bar{\mathbf{4}}$. The transverse space is a Ricci-flat Kähler manifold. Therefore there exists a covariantly constant spinor, and a general spinor field can be expanded in terms of the "one-particle states" $\Gamma_i \eta_0$, "two-particle states" $\Gamma_i \Gamma_j \eta_0$, and so on, plus their complex conjugates. In the case of negative chirality spinors $S, \tilde{S}$ this decomposition defines the worldsheet spinors as follows:

$$\begin{aligned} S &= \psi_-^i \Gamma_i \eta_0 + \bar{\psi}_-^{\bar{i}} \Gamma_{\bar{i}} \eta_0^* \\ \tilde{S} &= \psi_+^i \Gamma_i \eta_0 + \bar{\psi}_+^{\bar{i}} \Gamma_{\bar{i}} \eta_0^*, \end{aligned} \tag{2.11}$$

where η_0 is a covariantly constant spinor annihilated by all $\Gamma_{\bar{i}}$.

One can explicitly derive the action (2.10) in the manner of [24], where in the case of a non-constant R-R three-form field F_3, a non-supersymmetric theory has been obtained on the worldsheet. In our case the starting points are the background (2.7) and the light-cone gauge Green-Schwarz lagrangian when R-R fields are present, which can be found in [24, 17]. The part of the metric proportional to $|\partial_i W|^2$ turns into an interacting term for the bosonic fields on the worldsheet. The part of the lagrangian connected to F_5 is proportional to $iF_{+i_1 i_2 i_3 i_4} \theta \Gamma^- \Gamma^{i_1 i_2 i_3 i_4} \tilde{\theta}$. Choosing a convenient representation of the gamma matrices to solve the light-cone gauge conditions $\Gamma^+\theta = \Gamma^+\tilde{\theta} = 0$, for $S, \tilde{S}$, and then using the decomposition in (2.11), the above term of the light-cone lagrangian turns into Yukawa terms on the worldsheet. Putting everything together, one then arrives at the action (2.10) when the forms $(1,3)$ and $(3,1)$ are turned on. The arbitrary holomorphic function W, which parametrizes the background, becomes the superpotential on the worldsheet.

A crucial question is whether the backgrounds reviewed in this section describe solutions of superstring theory. Any action in light-cone IIB GS formalism can be turned, by completing with a proper set of fields, into a theory which is classically $\mathcal{N} = (2,2)$

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superconformally invariant. If this new action is also $\mathcal{N}=(2,2)$ superconformally invariant at the quantum level, the original light-cone GS background describes a solution of superstring theory. It has been shown [11] by using the above procedure, known as the U(4) formalism, that the two classes of IIB backgrounds found in [10] are exact solutions of superstring theory when the transverse target space is flat.

3. Superstring theory with superpotential $z^3 - z$

We have seen that superstrings in the above backgrounds produce, in light-cone gauge, $\mathcal{N}=(2,2)$ supersymmetric interacting field theories on the worldsheet. In general the action of an $\mathcal{N}=2$ supersymmetric theory contains in addition to the part in (2.10) an extra term L_V which depends on a holomorphic Killing vector V. Including this term and setting $F^i = -\frac{1}{2}g^{i\bar{i}}\partial_{\bar{i}}\bar{W}(\bar{Z})$ by using the equations of motion, a general $\mathcal{N}=(2,2)$ supersymmetric lagrangian is given, in terms of the component fields of the superfields by:

$$\begin{aligned} L &= L_K + L_W + L_V \\ &= g_{i\bar{j}}(\partial_\tau Z^i \partial_\tau \bar{Z}^{\bar{j}} - \partial_\sigma Z^i \partial_\sigma \bar{Z}^{\bar{j}}) + i g_{i\bar{j}}\bar{\psi}_+^{\bar{j}}(\partial_\tau - \partial_\sigma)\psi_+^i + i g_{i\bar{j}}\bar{\psi}_-^{\bar{j}}(\partial_\tau + \partial_\sigma)\psi_-^i + \\ &\quad + \frac{i}{2}\partial_i\partial_j W(Z)\psi_+^i\psi_-^j + \frac{i}{2}\partial_{\bar{i}}\partial_{\bar{j}}\bar{W}(\bar{Z})\bar{\psi}_-^{\bar{i}}\bar{\psi}_+^{\bar{j}} - \frac{1}{4}g^{i\bar{j}}\partial_i W(Z)\partial_{\bar{j}}\bar{W}(\bar{Z}) - \\ &\quad - |\tilde{m}|^2 g_{i\bar{j}} V^i V^{\bar{j}} - \frac{i}{2}(g_{i\bar{i}}\partial_j V^i - g_{j\bar{j}}\partial_{\bar{i}}V^{\bar{j}})(\tilde{m}\bar{\psi}_-^{\bar{i}}\psi_+^j + \bar{\tilde{m}}\bar{\psi}_+^{\bar{i}}\psi_-^j)\,, \end{aligned} \tag{3.1}$$

where the last line represents the extra term L_V, and the superfields are expanded as usual: $\Phi^i = Z^i + \sqrt{2}\theta^+\psi_+^i + \sqrt{2}\theta^-\psi_-^i + 2\theta^+\theta^- F^i + \cdots$. Since we have closed strings the worldsheet is a cylinder. The periodicity of the Green-Schwarz fermions leads to periodic boundary conditions on the worldsheet fermions.

In general these two-dimensional theories are massive and, in our case, they are defined on a cylinder with periodic boundary conditions on all fields. If these theories on the worldsheet are integrable, it should be possible at least in principle to find the string spectrum. A simplification is to take a superpotential of the form $W = W_1(Z^1) + W_2(Z^2) + W_3(Z^3) + W_4(Z^4)$ so that we obtain four independent two-dimensional theories. We consider the massive integrable theory given by the superpotential $W = \lambda((Z^1)^3/3 - \delta^2 Z^1)$, where Z^1 is the superstring coordinate in the 1-complex direction and δ, λ real parameters. To find the eigenvalues of the string light-cone gauge hamiltonian we need to analyze the full worldsheet theory, while to find the supergravity part of the string spectrum we have to consider either the $\alpha' \to 0$ or $p_- \to 0$ limit of the worldsheet theory, which we will do in section 4. In 2d language, this integrable $\mathcal{N}=(2,2)$ supersymmetric field theory is a relevant perturbation of the first superconformal minimal model [29]. One can bosonize the fermions to obtain the ordinary sine-Gordon model at a special coupling where it is supersymmetric, but we will continue to use the manifestly-supersymmetric language.

In superstring theory we take the transverse space to be flat: $g_{i\bar{j}} = \delta_{i\bar{j}}$. As shown in [10], in flat transverse space we have a holomorphic vector field V, and it is related to $\varphi_{i\bar{j}}$ as:

$$V_i = -i\varphi_{i\bar{j}}\bar{Z}^{\bar{j}}\,, \qquad V_{\bar{j}} = i\varphi_{i\bar{j}}Z^i\,. \tag{3.2}$$

However, we take the $(2,2)$ forms zero, $\varphi_{i\bar{j}} = 0$, so that $V = 0$. For the above superpotential the metric and the anti-self-dual 4-form are:

$$\begin{aligned} ds^2 &= -2dx^+dx^- - 32\lambda^2(z^1z^1 - \delta^2)(\bar{z}^{\bar{1}}\bar{z}^{\bar{1}} - \delta^2)dx^+dx^+ + 2dz^id\bar{z}^{\bar{i}} \\ F_5 &= dx^+ \wedge \varphi_4 \quad \varphi_4 = (2\lambda z^1)dz^1 \wedge d\bar{z}^{\bar{2}} \wedge d\bar{z}^{\bar{3}} \wedge d\bar{z}^{\bar{4}} + (2\lambda\bar{z}^{\bar{1}})d\bar{z}^{\bar{1}} \wedge dz^2 \wedge dz^3 \wedge dz^4 \, , \end{aligned} \tag{3.3}$$

Since we take W to depend only on Z^1, the other theories in Z^2, Z^3, Z^4 are massless and free and they can be easily solved. For this part one has the usual light-cone gauge hamiltonian consisting of a kinetic part and an oscillatory part expressed in terms of rising and lowering bosonic and fermionic operators:

$$H_1 = \frac{1}{2|p_-|}\sum_\alpha p^\alpha p^\alpha + \frac{1}{\alpha'|p_-|}(N + \tilde{N})\, , \tag{3.4}$$

where the summation is over the free coordinates $\alpha = 2, 3, 4, 6, 7, 8$. The occupation numbers expressed in complex coordinates are:

$$\begin{aligned} N &= \sum_{n=1}^{\infty}\left(\alpha^{\bar{k}}_{-n}\alpha^k_n + \alpha^k_{-n}\alpha^{\bar{k}}_n + nd^{\bar{k}}_{-n}d^k_n + nd^k_{-n}d^{\bar{k}}_n\right), \\ \tilde{N} &= \sum_{n=1}^{\infty}\left(\tilde{\alpha}^{\bar{k}}_{-n}\tilde{\alpha}^k_n + \tilde{\alpha}^k_{-n}\tilde{\alpha}^{\bar{k}}_n + n\tilde{d}^{\bar{k}}_{-n}\tilde{d}^k_n + n\tilde{d}^k_{-n}\tilde{d}^{\bar{k}}_n\right). \end{aligned} \tag{3.5}$$

As usual for a free hamiltonian the ground state of H_1 is defined by:

$$\alpha^k_n|0\rangle = \alpha^{\bar{k}}_n|0\rangle = \tilde{\alpha}^k_n|0\rangle = \tilde{\alpha}^{\bar{k}}_n|0\rangle = d^k_n|0\rangle = d^{\bar{k}}_n|0\rangle = \tilde{d}^k_n|0\rangle = \tilde{d}^{\bar{k}}_n|0\rangle = 0 \qquad n > 0\, , \qquad k = 2, 3, 4\, .$$

To find the total light-cone string hamiltonian $H = H_1 + H_2$ we need to find the spectrum of the interacting part H_2, with lagrangian given by (3.1) with $W = \lambda\left(\frac{1}{3}(Z^1)^3 - \delta^2 Z^1\right)$ When $\lambda = 0$ the theory becomes the flat space type-IIB superstring. We are interested in the superstring theory in the general pp-wave background with $\lambda \neq 0$ when at least four space-time supersymmetries are preserved. The parameter λ can be absorbed into rescaling of x^+, x^- which corresponds to rescaling of light-cone gauge energy and $|p_-|$; so in fact λ can be set to a fixed nonzero value. Therefore the eigenvalue configuration of H depends on δ, but not λ (other than just rescaling of energy). This is different from the maximally supersymmetric pp-wave background where any particular value of the background parameter is not special due to rescaling of x^+, x^-. It is convenient to rewrite the corresponding two-dimensional $\mathcal{N} = (2,2)$ supersymmetric field theory in terms of canonical dimensionless fields. After re-scaling $Z \to \sqrt{2\pi\alpha'}Z, \psi \to \sqrt{2\pi\alpha'}\psi, \delta \to \sqrt{2\pi\alpha'}\gamma, \lambda \to \mu/\sqrt{2\pi\alpha'}$, we have γ dimensionless and μ a mass parameter. The action is then

$$\begin{aligned} S_2 &= \frac{1}{2}\int d\tau \int_0^{2\pi\alpha'|p_-|} d\sigma L_2 \\ L_2 &= \partial_\tau Z\partial_\tau\bar{Z} - \partial_\sigma Z\partial_\sigma\bar{Z} - \frac{1}{4}\mu^2(Z^2 - \gamma^2)(\bar{Z}^2 - \gamma^2) + \\ &\quad + i\bar{\psi}_+(\partial_\tau - \partial_\sigma)\psi_+ + i\bar{\psi}_-(\partial_\tau + \partial_\sigma)\psi_- + i\mu Z\psi_+\psi_- + i\mu\bar{Z}\bar{\psi}_-\bar{\psi}_+\, . \end{aligned} \tag{3.7}$$

On the infinite line, the low-lying energies of many 2d Lorentz-invariant field theories are written as $E = \sqrt{p^2 + m^2}$, where m is the mass of a particle in the theory, and p its momentum. If the particles are interacting, the energy of multi-particle states cannot be generically determined, but the qualitative behavior can be described in terms of particles. Because of the non-renormalization theorems in an $\mathcal{N} = (2,2)$ theory, the particle content of the theory (3.7), can be inferred directly from the superpotential [29, 30]. The infinite-volume spectrum is described in terms of kinks interpolating between the vacua; each kink is a supersymmetry doublet of fermion numbers $1/2$ and $-1/2$. In this model there are no breathers; but they exist in the super sine-Gordon case, which we will discuss later. BPS kinks have mass $m = |W(Z(\sigma \to \infty)) - W(Z(\sigma \to -\infty))|$ where $Z(\sigma \to \pm\infty)$ are the values of the bosonic field at spatial infinity for a kink in infinite volume. For the superpotential $W = \mu(Z^3/3 - \gamma^2 Z)$ in (3.7) the two vacua are at $Z = \pm\gamma$, so $m = 4\mu\gamma^3/3 = 2\lambda\delta^3/(3\pi\alpha')$.

On a compact space, such as the one we have with circumference $R = 2\pi\alpha'|p_-|$, finding the spectrum is a more difficult problem. Even if the kinds of particles in the spectrum are known, their mass depends on R; in Feynman diagram language this is because virtual particles can propagate "around the world" [31]. The spectrum thus depends on the boundary conditions. Here we take both the bosonic and fermionic fields to satisfy periodic boundary conditions $Z(\tau,\sigma) = Z(\tau,\sigma + R), \psi(\tau,\sigma) = \psi(\tau,\sigma + R)$. As mentioned in [25] and [10], besides considering strictly periodic boundary conditions one may also impose identifications in the space of field configurations. This will modify the spectrum of the theory. At this point we can consider two situations. One is given by taking the identification $Z \simeq -Z, \psi \simeq -\psi$ in the space of fields. This means that the transverse space in the string light-cone gauge formalism is $\mathbb{C}/\mathbb{Z}_2 \times \mathbb{C}^3$. The other situation to consider, and the one we deal with in most of this paper, is the one with no identifications in field configuration, and the transverse space in string theory is then $\mathbb{C}^4 \simeq \mathbb{R}^8$. The requirement of translation invariance along σ gives the restriction on physical states:

$$P_2|\psi_{\text{phys}}\rangle = \frac{N - \tilde{N}}{\alpha'|p_-|}|\psi_{\text{phys}}\rangle\,. \tag{3.8}$$

Finding the eigenvalues of the H_2 is equivalent to finding the energy levels of the $1+1$-dimensional field theory with lagrangian (3.7) on a circle of circumference R. In the world-sheet theory, the only two dimensionful quantities are the mass m of the kinks, and the size of the system R. On dimensional grounds, the energy can therefore be written in terms of a scaling function: $E = f(mR)/R$. This means in terms of string quantities, the excited-state energies of the interacting part H_2 of the light-cone hamiltonian are

$$E_2^{(i)} = \frac{1}{12\alpha'|p_-|} f^{(i)}\left(\frac{4}{3}\lambda\delta^3|p_-|\right) \tag{3.9}$$

where the scaling function $f^{(i)}$, with $i = 1, \ldots, \infty$, can be computed using the 1+1-dimensional integrability techniques. The above expression is valid for any $\mu \neq 0$ or in string scale $\lambda \neq 0$. Note that regardless of the coupling no breather states exist, i.e. no small fluctuation modes at the minima of the superpotential exist, as one would expect. However, in the point particle limit $\alpha' \to 0$ with λ, δ and p_- fixed, the worldsheet theory reduces to a free massive theory with mass $\lambda\delta$ (as $\mu \to 0, \gamma \to \infty$ with $\mu\gamma = \lambda\delta = fixed$). This mismatching between the existence and non-existence of the fluctuation modes at the minima of the superpotential in the $\mu \to 0$ case and $\mu \neq 0$ case respectively, signals that the two situations are infinitely far (in the renormalization-group sense). We will discuss more this issue in section 4.

On general grounds, we know the ground state of H_2 with periodic boundary conditions has zero energy $E_2^{(0)} = 0$. In this case the Witten index [32] is two; there are two bosonic ground states with $P_2 = 0$ and no fermionic zero modes. This is fairly obvious from the superpotential, whose bosonic part has two minima. Since $N = \tilde{N} = 0$ the ground state of the light-cone gauge hamiltonian $H = H_1 + H_2$ contains no oscillators from the free part H_1. Therefore the ground state energy of H is just the kinetic part $\frac{1}{2|p_-|}\sum_\alpha p^\alpha p^\alpha$. As usual there is a degeneracy associated with the fermionic zero modes from the free part H_1, but now there is also a bosonic degeneracy coming from interacting part H_2. However, in contrast with the flat space now there are fewer fermionic zero modes as there are no fermionic zero modes for the ground state of H_2. There are two ways to take the string's point particle limit: $p_- \to 0$ or $\alpha' \to 0$. It appears that here the two limits give different answers in contrast to flat space or the maximally supersymmetric pp-wave. In the first case when $p_- \to 0$ the expression (3.9) is valid in the limit $p_- \to 0$, all excited levels decouple and we are left with the two bosonic ground states discussed above. Physically these states correspond to a point particle fixed at either $x^1 = \delta, x^5 = 0$ or $x^1 = -\delta, x^5 = 0$, but free in the other transverse directions $x^2, x^3, x^4, x^6, x^7, x^8$. In the other point particle limit, $\alpha' \to 0$, we have a free massive theory on the worldsheet and we are left with an infinite number of states which do not decouple in the limit. Physically the string reduces to a point particle in a harmonic potential $(X^1)^2 + (X^5)^2$ at either $x^1 = \delta, x^5 = 0$ or $x^1 = -\delta, x^5 = 0$, but free in the other transverse directions. In the next section we will discuss the point particle string spectrum in more detail.

Finding the rest of the finite-size spectrum is a much more difficult problem. However, some progress has been made in integrable field theories by using the thermodynamic Bethe ansatz. Usually this technique is used to compute the free energy for a system on the infinite line at some non-zero temperature T. This amounts to computing the partition function on an infinitely-long cylinder of circumference $R = 1/T$, which yields the Casimir energy of the system in size R. With periodic boundary conditions around the cylinder, the Casimir energy here is exactly 0, as we have already indicated. In some cases, however, one can continue the thermodynamic Bethe ansatz equations to obtain the exact finite-size energies of excited states [14]. One ends up with a set of coupled non-linear integral equations whose solution yields the energies. For the above action S_2 with periodic boundary conditions, this computation has already been done for states with total momentum $P_2 = 0$ [15]. This analysis has been extended to find the R-dependence of single-particle states as well [33]; these one-particle states can appear when one takes the identifications in transverse target space $Z \simeq -Z, \psi \simeq -\psi$. In what follows we consider no identifications in the transverse space.

Since the analysis of [15] is quite technical, we do not reproduce it here, but refer the reader there for details. In the $mR \to 0$ limit, the energy levels of the states found in [15] are $E_2^{(j)} = \frac{\pi}{6R}\left[(2j+1)^2 - 1\right]$, $j = 1 \ldots \infty$. In the limit when mR is very large, the particles

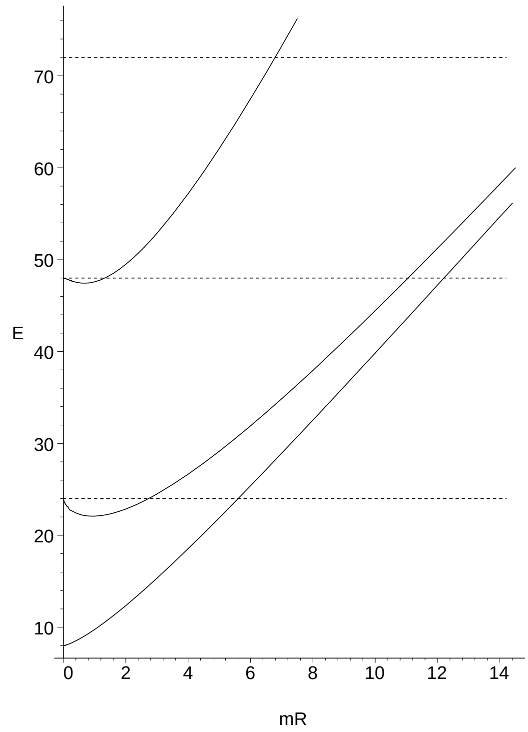

Figure 1: The first three excited-state energies of H_2: $E_2^{(1)}, E_2^{(2)}, E_2^{(3)}$ represented by continuous lines. The first three oscillator levels of H_1 represented by dashed lines. The energies are in units of $\frac{1}{12\alpha'|p_-|}$, while mR in string scale is $\frac{4}{3}\lambda\delta^3|p_-|$.

are far apart and effectively free. Because of the periodic boundary conditions, the energy levels correspond to states with the same number of kinks and antikinks, each of mass m. For example, the first excited level in the large mR limit consists of two free particles with momenta $\pm\pi/R$ and total energy $E_2^{(1)} \approx 2\sqrt{m^2 + (\pi/R)^2}$. The second excited state $E_2^{(2)}$ also represents two free particles, but now with momenta $\pm 2\pi/R$. In the large-mR limit, it was shown in [15] that the energies $E_2^{(3k+1)}$ and $E_2^{(3k+2)}$, $k = 0 \ldots \infty$ approach each other; they correspond to states with $2k+1$ kinks and $2k+1$ antikinks. The energies $E_2^{(3k)}$, $k = 1 \ldots \infty$ correspond to states with $2k$ kinks and $2k$ antikinks. These do not comprise all excited-state energy levels, but it seems likely that the techniques of [15, 33] could be extended to others if desired. Away from large mR limit, one must take into account the interactions between the kink and antikink. We have numerically solved the non-linear integral equations found in [15] for $E_2^{(1)}, E_2^{(2)}, E_2^{(3)}$, and displayed the results in figure 1.

We consider only excited states of the two-dimensional interacting theory which have zero momentum $P_2 = 0$, so that the physical states of light-cone string hamiltonian H satisfy $N = \tilde{N}$. Apart from the continuous part $\frac{1}{2|p_-|}\sum_\alpha p^\alpha p^\alpha$ the light-cone gauge hamiltonian has a discrete spectrum. On the same graph (figure 1) viewed for a constant value of R we can also represent the oscillatory part of H_1 depending only on R, namely $\frac{2N}{\alpha'|p_-|}$ for $N = 1, 2, \ldots$. Thus all eigenvalues of the light-cone string hamiltonian H can be obtained. They depend on the background inputs λ, δ and on the string parameter $|p_-|$. The eigenvalue configuration depends non-trivially on $mR = \frac{4}{3}\lambda\delta^3|p_-|$. The intersections of $\frac{2N}{\alpha'|p_-|}$ with the eigenvalues of H_2 gives sectors in mR; in two different such sectors the eigenvalue configuration is different. For example, apart from the non-oscillatory part of H_1, the first six levels of H for $2.7 < mR < 5.6$ are:

$$\begin{aligned} E^{(0)} &= 0\,, & E^{(1)} &= E_2^{(1)}\,, & E^{(2)} &= \frac{2}{\alpha'|p_-|}\,, \\ E^{(3)} &= E_2^{(2)}\,, & E^{(4)} &= E_2^{(1)} + \frac{2}{\alpha'|p_-|}\,, & E^{(5)} &= \frac{4}{\alpha'|p_-|} \end{aligned} \tag{3.10}$$

while for $5.6 < mR < 11.1$ they are:

$$\begin{aligned} E^{(0)} &= 0\,, & E^{(1)} &= \frac{2}{\alpha'|p_-|}\,, & E^{(2)} &= E_2^{(1)}\,, \\ E^{(3)} &= E_2^{(2)}\,, & E^{(4)} &= \frac{4}{\alpha'|p_-|}\,, & E^{(5)} &= E_2^{(1)} + \frac{2}{\alpha'|p_-|}\,. \end{aligned} \tag{3.11}$$

Due to the fact that $g_{++} \neq 0$ the string feels a gravitational force that pulls it to the regions where $-g_{++}$ is minimum, namely the regions with $z^1 = \pm\delta$ ($x^1 = \pm\delta, x^5 = 0$). Then kink-antikink pairs on the worldsheet going between minima $x^1 = \pm\gamma, x^5 = 0$ correspond to a string that goes between the two positions where $-g_{++}$ has a minimum in target space, namely $\pm\delta$.

As the (infinite-volume) kink mass m gets bigger due to changes in background parameters λ, δ, the oscillatory states of H_1 have lower energy than the massive states of H_2. As $m \to \infty$ with p_- fixed, the states of H_2 effectively decouple. In the string scale, this limit corresponds to $\delta \to \infty$ and/or to a very strong R-R field $\lambda \to \infty$. Therefore in this limit the transverse space seen by the string in its excited states is six-dimensional and it consists in two hyperplanes sitting at $x^1 = \delta, x^5 = 0$ and $x^1 = -\delta, x^5 = 0$. In this limit the string is highly excited in the free directions, and it is in the ground state in the interacting directions. Alternatively, we can keep the background fixed, i.e. λ, δ fixed. Then the above limit is obtained for strings with very large $|p_-|$, which is the IR limit of the worldsheet theory. It seems possible that in this limit it might be a correspondence between the string theory under discussion and a string theory on a partially discrete target space, e.g. a transverse target space $\mathbb{R}^6 \times \{-\delta, \delta\} \times \{0\}$. Because of the UV/IR relationship between worldsheet and target space scales, the above discrete space appears at very short space-time scales.

4. Supergravity part of the superstring spectrum

We now return to the low-energy states of H and analyze the supergravity part of superstring, the "massless" string modes. The supergravity part of string spectrum comes from all possible states giving light-cone energies which do not decouple in the limit $\alpha' \to 0$ while keeping everything else fixed. As we already mentioned, in the $\alpha' \to 0$ limit the worldsheet theory becomes a free massive theory. In this case, as in the case of the maximally supersymmetic pp-wave, we expect to obtain the usual type-IIB supergravity spectrum. As we have seen, the string's point particle limits $\alpha' \to 0$ and $p_- \to 0$ give different answers. In the first case we obtain the IIB supergravity spectrum, but in the second case some of these modes are missing, as we will show in this section.

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In the remaining of this section we analyze the supergravity spectrum obtained in the limit $p_- \to 0$, which means the worldsheet radius $R \to 0$, while everything else is kept fixed. As opposed to the case of a free massive superfield on the world sheet discussed in [4, 5], only a few modes for the the theory discussed in section 3 remain in the $p_- \to 0$ limit. To explain this issue a little further, we amplify on a discussion in [10]. A free massive superfield of mass M on the world sheet of circumference R results from the superpotential $Mz^2/2$. The energy levels are are $\sqrt{(k_n)^2 + M^2}$, where $k_n = 2n\pi/R$ is the world-sheet momentum. Thus in the $R \to 0$ limit, all of these modes have very large energy except for the zero-momentum modes with $n = 0$. For a free massive superfield, there are an infinite number of these, because one can have as many zero-momentum bosonic modes present as desired.

The question to ask is then: what happens to these zero-momentum modes when interactions are included? It is useful to answer this question first in the case of super sine-Gordon, where the superpotential is $W = (M/\beta^2)\cos(\beta z)$, with M a mass parameter and β the usual coupling. In the limit $\beta \to 0$, this reduces to the free superfield, and we have an infinite number of zero modes. This fits in perfectly with the results from integrable field theory: in the limit $\beta \to 0$ there are an infinite number of "breather" states in the spectrum, which have masses $M, 2M, 3M, \ldots$. The kinks have mass proportional to $1/\beta$ in this limit, and so decouple. Thus a state with a single breather of momentum zero will have energy jM for some integer j. All of these states precisely reproduce the spectrum of a free superfield (the analogous statement for the ordinary sine-Gordon model is that $\beta \to 0$ spectrum is that of a single free boson).

Since the super-sine-Gordon model is integrable for any β, we can understand what happens to these states as one increases β. What is known is that as β is increased, the breather states drop out of the spectrum. This does not mean that there are discontinuities in the energy levels as a function of β — it means that a (single-particle) breather state turns into a (two-particle) kink-antikink state as β is increased. One can see this explicitly in the Bethe-ansatz solution of the ordinary sine-Gordon/massive Thirring model [34]. For large enough β ($\beta^2 = 8\pi$ in the usual normalization of super-sine-Gordon), *all* of the breather states have dropped out of the theory. For ordinary sine-Gordon, this point is where the sine-Gordon model is equivalent to a free Dirac fermion, so we can easily see what has happened to the zero-momentum single-particle breather states: they correspond to a fermion and antifermion (kink and antikink in the sine-Gordon language) with opposite momentum. The crucial point is that because here the excitations are fermions, there is only one state where both particles have zero momentum: the state with one fermion of zero momentum and one antifermion of zero momentum. This state can be identified with the breather of lowest mass. The higher zero-momentum breather states must therefore be identified with a state with fermion and antifermion, but where the two particles have non-zero individual momentum. In a finite-size system, these states must have energy proportional to $1/R$, and so disappear in the small-R limit. It would be quite interesting to confirm this by extending the computation of [33] to compute the energy of the single-particle breather states in finite volume.

In the super-sine-Gordon case, the same thing must happen — only two states of relative and total momenta zero remain at $\beta^2 = 8\pi$. (It is two here because there is a supersymmetric doublet of kinks of charge $\pm 1/2$ [35]; there are therefore two two-particle states of charge 0 and individual momenta zero.) This follows from the fact that in all theories solvable by the Bethe ansatz, the excitations are fermionic in that only one excitation of a given type is allowed per level (for example, in ordinary sine-Gordon, one can only have a single zero-momentum breather of a given type present). For super-sine-Gordon, this fact was confirmed by the thermodynamic Bethe ansatz computation of [35]. Therefore, all but two of the zero-charge zero-momentum states present as $\beta \to 0$ must have energy proportional to $1/R$ (possibly multiplied by a logarithm) by the time β^2 reaches 8π.

We have therefore argued that although there are an infinite number of low-energy states in the limit of free particles on world sheet, i.e. $\beta \to 0$ limit, only a small number of states survive for strong enough interactions in the case of super sine-Gordon. Let us see what happens when considering the superpotential $z^3 - z$ studied above. In the small-R limit, i.e. the UV limit on the worldsheet, the spectrum is that of the conformal field theory with superpotential μz^3, or in string scale λz^3; with periodic boundary conditions this theory has only two ground states. All other states will have energy proportional to $1/R$. Another way of seeing how this can happen is by deforming the free massive theory: if one adds a superpotential z^3 to the z^2, one obtains the $z^3 - z$ theory (the two are equivalent by a constant shift in z). However, the conformal field theory with μz^3 is infinitely far (in the renormalization-group sense) from the free massive superfield; this must be so because the theories have different Witten indices [36]. This means that the conformal field theory with superpotential μz^3 cannot be reached by starting with a free massive superfield and adding small perturbations.[1] Thus it is not surprising that all but a finite number of low-energy states go away in the small R-limit.

In fact, generically any worldsheet theory can effectively be described in the $R \to 0$ limit by a conformal field theory. Because the only mass scale here is $1/R$, all states other than exact ground states will have high energy. If the conformal theory consists of free superfields, then the corresponding supergravity spectrum will be the usual one. If the model is solvable by the Bethe ansatz, then states will drop out of the spacetime supergravity spectrum unless the worldsheet spectrum contains an infinite number of particles. Moreover, it seems likely generically, that states will drop out unless the interactions in the worldsheet conformal field theory are weak. To give a well-known example, in Calabi-Yau compactification, the world-sheet theory is a sigma model with a Calabi-Yau manifold as a target space; this can change the low-energy spectrum dramatically.

We can see the effects of these considerations on our full theory by examining the fermion zero-mode spectrum explicitly. The ground state has a degeneracy coming from the two bosonic states of the interacting 2d theory. In addition there is the degeneracy from the algebra satisfied by the zero modes of the light-cone GS fermions S and $\tilde{S}$ both having the same chirality, which we take to be negative. We need to find an irreducible

[1] In [36] it is argued that a Landau-Ginzburg theory given by a large class of superpotentials flows in the UV limit to a free massless superfield. However, the free theory is reached asymptotically, which means that in the UV region the theory is not equivalent with a free CFT plus a small perturbation. The superpotential does not go away completely even in the far UV, which is a sign that non-perturbative effects remain in the limit.

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representation of this algebra. As we already mentioned there are no fermionic zero modes in the interacting directions, and the other worldsheet fermionic zero modes satisfy:

$$\begin{aligned}\left\{d_0^i, d_0^j\right\} &= \left\{d_0^{\bar{i}}, d_0^{\bar{j}}\right\} = \left\{\tilde{d}_0^i, \tilde{d}_0^j\right\} = \left\{\tilde{d}_0^{\bar{i}}, \tilde{d}_0^{\bar{j}}\right\} = 0 \\ \left\{d_0^i, d_0^{\bar{j}}\right\} &= \left\{\tilde{d}_0^i, \tilde{d}_0^{\bar{j}}\right\} = \delta^{i\bar{j}}\,, \qquad i,j = 2,3,4\,. \end{aligned} \tag{4.1}$$

The gamma matrices in complex coordinates satisfy $[\Gamma_i, \Gamma_{\bar{i}}] = 2\delta_{i\bar{i}}$. As usual in complex space we also define $\Gamma_{i\bar{i}} = \frac{1}{2}(\Gamma_i\Gamma_{\bar{i}} - \Gamma_{\bar{i}}\Gamma_i)$, with $i = 1,2,3,4$. They satisfy:

$$[\Gamma_{i\bar{i}}, \Gamma^{\bar{k}}] = 2\Gamma_i\delta_{\bar{i}}^{\bar{k}}\,, \qquad [\Gamma_{i\bar{i}}, \Gamma^k] = -2\Gamma_{\bar{i}}\delta_i^k$$

A constant spinor annihilated by all $\Gamma_{\bar{i}}$ can be taken to be $\eta_0 = \frac{i}{2}(-1,-1,-1,-1)$. Then

$$\Gamma_1\eta_0 = \frac{i}{2}(1,-1,-1,-1)\,, \qquad \Gamma_{\bar{1}}\eta_0^* = \frac{i}{2}(-1,1,1,1) \tag{4.3}$$

and similar expressions for the other actions of Γ_i and $\Gamma_{\bar{i}}$ on η_0, η_0^*. Using (2.11) one obtains the GS fermions S and $\tilde{S}$ in terms of the worldsheet fermions. The light-cone gauge superstring lagrangian L, taken in the string scale, is then

$$\begin{aligned} L = &\frac{1}{2}\left(\sum_{i=1}^{8}\partial_+X^i\partial_-X^i\right) - \frac{\lambda^2}{4}\left[\frac{1}{4}\left((X^1)^2+(X^5)^2\right)^2 + \delta^4 - \delta^2\left((X^1)^2-(X^5)^2\right)\right] + \\ &+\frac{i}{4}\left(S^a\partial_+S^a + \tilde{S}^a\partial_-\tilde{S}^a\right) - \frac{i\lambda}{8\sqrt{2}}\left(X^1S^aM_{ab}\tilde{S}^b - X^5S^aP_{ab}\tilde{S}^b\right)\,, \end{aligned} \tag{4.4}$$

where $\partial_\pm = \partial_\tau \pm \partial_\sigma$, and matrices M and P are:

$$M = \begin{pmatrix} Q & 0 \\ 0 & -Q \end{pmatrix} \qquad P = \begin{pmatrix} 0 & Q \\ Q & 0 \end{pmatrix}, \quad \text{with } Q = \begin{pmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{pmatrix}. \tag{4.5}$$

The background (3.3) in real coordinates becomes:

$$\begin{aligned} ds^2 = &-2dx^+dx^- - 32\lambda^2\left[\frac{1}{4}\left((x^1)^2+(x^5)^2\right)^2 + \delta^4 - \delta^2\left((x^1)^2-(x^5)^2\right)\right]dx^+dx^+ \\ &+\sum_{i=1}^{8}dx^idx^i \\ \varphi_4 = &\frac{\lambda}{\sqrt{2}}(x^1dx^1 - x^5dx^5)(dx^2\wedge dx^3\wedge dx^4 - dx^2\wedge dx^7\wedge dx^8 - dx^4\wedge dx^6\wedge dx^7 + \\ &+ dx^3\wedge dx^6\wedge dx^8) + \frac{\lambda}{\sqrt{2}}(x^5dx^1 + x^1dx^5)\times \\ &\times(dx^2\wedge dx^3\wedge dx^8 - dx^2\wedge dx^4\wedge dx^7 + dx^3\wedge dx^4\wedge dx^6 - dx^6\wedge dx^7\wedge dx^8)\,. \end{aligned} \tag{4.6}$$

We see that the metric and the pure bosonic part of the light-cone gauge lagrangian L have an SO(6) global symmetry, and the pure fermionic part an SO(8) symmetry. But both the bosonic-fermionic part of L and the 4-form φ_4 have a global symmetry given by a group

$G = \mathrm{SO}(3)$ which rotates the coordinates $2, 3, 4$ the same way as the coordinates $6, 7, 8$. Note that this is different from the usual maximally supersymmetric pp-wave where the global symmetry of the light-cone gauge lagrangian is $\mathrm{SO}(4) \times \mathrm{SO}(4)$, or the flat space with little group $\mathrm{SO}(8)$.

Let us return now to zero modes. Using the zero modes anti-commutation relations of the worldsheet fermions (4.1) we obtain the Clifford algebra satisfied by the GS fermions zero modes:

$$\left\{S_0^a, S_0^b\right\} = \left\{\tilde{S}_0^a, \tilde{S}_0^b\right\} = \frac{1}{2}A^{ab}\,, \qquad \left\{S_0^a, \tilde{S}_0^b\right\} = 0 \qquad a, b = 1, 2, \ldots, 8 \tag{4.7}$$

where the 8×8 matrix A has the form:

$$A = \begin{pmatrix} B & 0 \\ 0 & B \end{pmatrix}, \tag{4.8}$$

with

$$B = \begin{pmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & -1 & -1 \\ 1 & -1 & 3 & -1 \\ 1 & -1 & -1 & 3 \end{pmatrix}. \tag{4.9}$$

This is different than the corresponding algebra in the usual IIB superstring ($\lambda = 0$) which we would have obtained having all worldsheet fermionic zero modes present, corresponding to replacing A_{ab} in (4.7) with $4\delta^{ab}$. The rank of the matrix A in (4.7) is only six, which means that the Clifford algebra (4.7) is degenerate. To find an irreducible representation of the Clifford algebra (4.7) let us consider an $\mathrm{SO}(8)$ rotation given by R which changes the spinors $S, \tilde{S}$ into $\chi, \tilde{\chi}$:

$$R = \begin{pmatrix} 0 & C \\ -C & 0 \end{pmatrix}, \qquad \text{with } C = \frac{1}{2}\begin{pmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{pmatrix}. \tag{4.10}$$

In this basis the fermionic zero modes satisfy the algebra:

$$\left\{\chi_0^a, \chi_0^b\right\} = \left\{\tilde{\chi}_0^a, \tilde{\chi}_0^b\right\} = 2D^{ab}\,, \qquad \left\{\chi_0^a, \tilde{\chi}_0^b\right\} = 0 \qquad a, b = 1, 2, \ldots, 8 \tag{4.11}$$

with

$$D = \begin{pmatrix} 0 & & & \\ & I_3 & & \\ & & 0 & \\ & & & I_3 \end{pmatrix}, \tag{4.12}$$

where I_3 is the 3×3 identity matrix. As usual we can form fermionic oscillators:

$$b_a = \frac{\chi_0^a + i\tilde{\chi}_0^a}{\sqrt{2}}\,, \qquad b_a^+ = \frac{\chi_0^a - i\tilde{\chi}_0^a}{\sqrt{2}} \qquad a = 1, \ldots, 8\,. \tag{4.13}$$

They satisfy the anti-commutation relations:

$$\left\{b_a, b_b^+\right\} = D_{ab}\,, \qquad \left\{b_a, b_b\right\} = \left\{b_a^+, b_b^+\right\} = 0\,. \tag{4.14}$$

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We therefore obtain 6 independent fermionic oscillators, giving a representation containing a number of *64* states, *32* bosonic *32* fermionic. Due to the fact that there are two bosonic states coming from the 2d lagrangian L_2, the total number of states of the supergravity part of superstring spectrum is *128*, half of them bosonic, half fermionic states. Under the "little group" G defined above we can classify the *64* states coming from the Clifford algebra. Since G contains elements (rotations) acting identically on both the set of coordinates $2,3,4$ and the set $6,7,8$, we do not have space-time fields indexed by indices belonging to different sets (mixed indices), as we would have for a group $\mathrm{SO}(3) \times \mathrm{SO}(3)$. The surviving type-IIB supergravity modes have the space-time field interpretation under the group G as:

graviton:	$g_{ij}(5)$ $g_{i'j'}(5)$
complex scalar field:	$\phi(2)$
complex two-form field:	$b_{ij}(6)$ $b_{i'j'}(6)$
four-form field:	$a_{ij}(3)$ $a_{i'j'}(3)$ a_1 a_2
spin 1/2 field:	$\lambda_1(4)$ $\lambda_2(4)$
spin 3/2 field:	$\psi_i(12)$ $\psi_{i'}(12)$

where $i,j,k = 2,3,4$ and $i',j',k' = 6,7,8$, and in brackets we indicate the number of independent components. The two $\lambda's$ are complex two-component spinors of $\mathrm{SO}(3)$. From a general four-form field of IIB supergravity $a_{\mu\nu\alpha\beta}$ $(\mu,\nu,\alpha,\beta = 1,\ldots,8)$ less components can survive under G. A mixed component $a_{i';ijk}$ can have non-vanishing components from the internal symmetry of indices i,j,k giving the above antisymmetric tensor a_{ij}. The same is valid for $a_{i;i'j'k'}$ giving the term $a_{i'j'}$. The other mixed components $a_{ij;i'j'}$ give two real scalar fields a_1, a_2. The total number of *128* states of the supergravity part of the string spectrum is represented by two such sets of fields in space-time.

In general the space-time interpretation of the supergravity part of the string spectrum is that it is isomorphic with the fluctuation modes of type-IIB supergravity fields expanded near the plane-wave type background given in (3.3). Solving analytically the equations satisfied by the above fluctuations is impossible because they are non-linear. However, there are in total only 128 supergravity modes in the superstring spectrum instead of the usual 256 IIB supergravity modes. It appears that some of the IIB supergravity modes are suppressed when we obtain the supergravity part of the string spectrum by taking the limit $p_- \to 0$, i.e UV limit of the worldsheet theory. This situation is as if the target space is effectively compactified on a Calabi-Yau manifold. It would be interesting to find such a compactified string theory dual to the one we considered above.

5. Other backgrounds giving integrable theories on the worldsheet

5.1 Superstring theories with more than one interacting direction

We can also consider superstring theories of the above type with deformed cubic superpotentials but with 2,3 or 4 interacting 2d quantum field theories. Consider the superpotential:

$$W = \lambda_1 \left(\frac{1}{3}(Z^1)^3 - \delta_1^2 Z^1 \right) + \lambda_2 \left(\frac{1}{3}(Z^2)^3 - \delta_2^2 Z^2 \right). \tag{5.1}$$

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The background in real coordinates is:

$$
\begin{aligned}
ds^2 &= -2dx^+dx^- - 32\lambda_1^2\left[\frac{1}{4}\left((x^1)^2+(x^5)^2\right)^2+\delta_1^4-\delta_1^2\left((x^1)^2-(x^5)^2\right)\right]dx^+dx^+ - \\
&\quad -32\lambda_2^2\left[\frac{1}{4}\left((x^2)^2+(x^6)^2\right)^2+\delta_2^4-\delta_2^2\left((x^2)^2-(x^6)^2\right)\right]dx^+dx^+ + \sum_{i=1}^{8}dx^idx^i \\
\varphi_4 &= \left(\frac{\lambda_1x^1-\lambda_2x^2}{\sqrt{2}}\right)(dx^1\wedge dx^2+dx^5\wedge dx^6)(dx^3\wedge dx^4-dx^7\wedge dx^8) - \\
&\quad -\left(\frac{\lambda_1x^5-\lambda_2x^6}{\sqrt{2}}\right)(dx^5\wedge dx^2-dx^1\wedge dx^6)(dx^3\wedge dx^4-dx^7\wedge dx^8) + \\
&\quad +\left(\frac{\lambda_1x^5-\lambda_2x^6}{\sqrt{2}}\right)(dx^1\wedge dx^2+dx^5\wedge dx^6)(dx^7\wedge dx^4+dx^3\wedge dx^8) + \\
&\quad +\left(\frac{\lambda_1x^1-\lambda_2x^2}{\sqrt{2}}\right)(dx^5\wedge dx^2-dx^1\wedge dx^6)(dx^7\wedge dx^4+dx^3\wedge dx^8)\,. \qquad (5.2)
\end{aligned}
$$

The global symmetry of this background is given by a group $G_1 = \mathrm{SO}(2)$ which rotates the coordinates $3,4$ the same way as the coordinates $7,8$.

On the worldsheet the above background gives two interacting 2d theories in Z^1, Z^2 and free theories in Z^3, Z^4. For each of the interacting theories the exact energy eigenvalues can be computed and represented as in figure 1. Therefore the full energy eigenvalue configuration of the light-cone gauge string hamiltonian can be obtained as depending on two dimensionless parameters $\frac{4}{3}\lambda_1\delta_1^3|p_-|$ and $\frac{4}{3}\lambda_2\delta_2^3|p_-|$. As in the previous sections the ground state of the light-cone gauge string hamiltonian contains no excited states from the interacting parts and no oscillatory states from the free parts. Now there are fewer fermionic zero modes than in the previous section but there are four bosonic states. By proceeding as in section 4 we can find the "massless" string states. Due to the degeneracy of the zero-modes GS fermions we have *16* states in space-time, half of them bosonic, half fermionic states. The surviving states of IIB supergravity can be classified in space-time under the group G_1 as:

graviton:	$g_{ij}(2)$	$g_{i'j'}(2)$
antisymmetric tensor:	$b_{ij}(1)$	$b_{i'j'}(1)$
complex scalar field:	$\phi(2)$	
spin 1/2 field:	$\lambda_1(2)$	$\lambda_2(2)$
spin 3/2 field:	$\psi_1(2)$	$\psi_2(2)$

where $i,j = 3,4$ and $i',j' = 7,8$, and the two $\lambda's$ are complex one-component Weyl spinors of SO(2). Since the bosonic part of H has a fourfold degeneracy we have a total of *64* "massless" string states represented by four sets of fields each containing the above fields.

We may take now the superpotential of the form:

$$
W = \sum_{i=1}^{3}\lambda_i\left(\frac{1}{3}(Z^i)^3-\delta_i^2Z^i\right). \qquad (5.3)
$$

By similar procedures as before we see that there are 4 states from the Clifford algebra of fermionic zero modes. In this case the global symmetry of the background is $\mathrm{SO}(1)\times\mathrm{SO}(1)$. Under this group we can have only a complex scalar field ϕ and two real one-component $\mathrm{SO}(1)$ spinors λ_1, λ_2. Since the bosonic degeneracy is now 8, the total number of "massless" string states in space-time is *32* (half bosonic, half fermionic states). There are eight sets of fields each consisting in a complex scalar field and two spinors.

Lastly we can consider the superpotential:

$$W = \sum_{i=1}^{4} \lambda_i \left(\frac{1}{3}(Z^i)^3 - \delta_i^2 Z^i \right). \tag{5.4}$$

Now there are no free parts in the light-cone gauge hamiltonian. Also, for the ground state there is no fermionic degeneracy, and therefore there is a total of *16* bosonic "massless" string states which correspond to *16* real scalar fields in space-time.

As in section 4 in all the above cases the supergravity part of superstring spectrum, obtained by taking the limit $p_- \to 0$, appears not to be in one-to-one correspondence to type-IIB supergravity modes, as some of these modes are suppressed. However, in the limit $\alpha' \to 0$ we obtain the full IIB supergravity spectrum in all the above cases.

5.2 Superstring theory on backgrounds giving a $\mathcal{N} = (2,2)$ supersymmetric sine-Gordon

One can also consider the superstring theory obtained by choosing the superpotential $W = \frac{M}{\omega^2}\cos(\omega Z^1)$. The background in this case is given by:

$$\begin{aligned} ds^2 &= -2dx^+dx^- - 32\left|\frac{M}{\omega}\sin(\omega z^1)\right|^2 dx^+dx^+ + 2dz^i d\bar{z}^{\bar{i}} \\ F_5 &= dx^+ \wedge \varphi_4\,, \qquad \varphi_4 = -M\cos(\omega z^1)dz^1 \wedge d\bar{z}^{\bar{2}} \wedge d\bar{z}^{\bar{3}} \wedge d\bar{z}^{\bar{4}} + \text{c.c.} \end{aligned} \tag{5.5}$$

Again we can rewrite the worldsheet action in terms of dimensionless fields by rescaling the fields and parameters ($Z^1 \equiv Z$): $Z \to \sqrt{2\pi\alpha'}Z$; $\omega \to \frac{\beta}{\sqrt{2\pi\alpha'}}$. The interacting part of light-cone gauge action is given by:

$$\begin{aligned} S_2 &= \frac{1}{2}\int d\tau \int_0^{2\pi\alpha'|p_-|} d\sigma L_2 \\ L_2 &= \partial_\tau Z \partial_\tau \bar{Z} - \partial_\sigma Z \partial_\sigma \bar{Z} - \frac{M^2}{4\beta^2}|\sin\beta Z|^2 + i\bar{\psi}_+(\partial_\tau - \partial_\sigma)\psi_+ + i\bar{\psi}_-(\partial_\tau + \partial_\sigma)\psi_- - \\ &\quad - \frac{i}{2}M\cos(\beta Z)\psi_+\psi_- - \frac{i}{2}M\cos(\beta\bar{Z})\bar{\psi}_-\bar{\psi}_+\,, \end{aligned} \tag{5.6}$$

In the weak coupling limit when $\beta \to 0$ or in string scale $\omega \to 0$, one can keep the first terms in the expansion of the superpotential and obtain $W = \frac{MZ^2}{2}$. For this superpotential the equations of motion of the light-cone gauge lagrangian can be solved analytically. In terms of real coordinates and GS spinors $S, \tilde{S}$, the light-cone gauge lagrangian density L contains two free massive scalars (X^1, X^5), six free massless scalars while all the fermions are massive and free. This can be solved exactly in a similar way as in the case of the usual pp-wave background. Thus, in the limit $\beta \to 0$, the supergravity modes of the string spectrum are the IIB supergravity modes, as expected for weak coupling. Further, as in the case of maximally supersymmetric pp-wave, the IIB supergravity spectrum is in one-to-one correspondence with the IIB supergravity fluctuation modes around the background under consideration.

Let us consider now the supergravity limit $\alpha' \to 0$ while everything else is fixed (M, ω, p_-). In this limit the worldsheet theory reduces to a free massive theory (like in the case of $z^3 - z$ superpotential) with mass M, as $\beta \to 0$ in the $\alpha' \to 0$ limit. As we already mentioned at the beginning of section 4, from the worldsheet point of view we have, in the limit $\beta \to 0$, an infinite number of breather states with masses $M, 2M, 3M, \ldots$. As in the case of the superpotential $z^3 - z$ we obtain the full IIB supergravity spectrum in the limit $\alpha' \to 0$. The supergravity part of the string spectrum has the hamiltonian:

$$H_0 = \frac{1}{2|p_-|} \sum_\alpha p^\alpha p^\alpha + fM \,, \tag{5.7}$$

where f is an integer number depending on the level of excitation of the harmonic oscillators, and α runs over the free coordinates $2, 3, 4, 6, 7, 8$. Let us now consider solving the scalar fluctuation Φ belonging to the massless supergravity multiplet. The curved-space Klein-Gordon equation satisfied by this fluctuation $\Box\Phi = 0$ has been considered in [24]. It has been shown that the light-cone gauge hamiltonian corresponding to the supergravity part of superstring spectrum is given by:

$$H_0 = \frac{1}{2|p_-|} \sum_\alpha p^\alpha p^\alpha + \frac{1}{2|p_-|}(a_r + e_n), \tag{5.8}$$

where the discrete set of values a_r and e_n depend on M, ω, and can be determined by solving two differential equations having solutions expressed in terms of Mathieu functions. For arbitrary values of M, ω this hamiltonian does not match the "massless" string states hamiltonian (5.7). Therefore it seems that the IIB supergravity spectrum is not in one-to-one correspondence to the IIB supergravity fluctuation modes around the background under consideration unless $\omega \to 0$, which reduces to the above disscused weak coupling limit $\beta \to 0$.

Let us consider what happens when we take the limit $p_- \to 0$ while keeping everything else fixed ($\beta \neq 0$). For $\beta^2 < 8\pi$ the spectrum of the worldsheet theory consists of kinks, antikinks and a finite number of breathers. We, however, do not have the breather state energies. As we mentioned in section 4 computing these energies as functions of β and the worldsheet size R would be interesting. The regime where we can analyze the behavior of the energy levels in the limit $p_- \to 0$ is for $\beta^2 \geq 8\pi$. In this case, when considered on two dimensional non-compact space, the supersymmetric sine-Gordon theory has a spectrum consisting of only kinks and anti-kinks with masses $m = \frac{2M}{\beta^2}$, and no stable breathers (kink-antikink bound states). When the direction x^1 is compact, with identification $x^1 \cong x^1 + 2\pi/\omega$., the kinks and anti-kinks interpolate between the two supersymmetric vacua at $x^1 = 0, x^5 = 0$ and $x^1 = \pi/\beta, x^5 = 0$, or in string scale at $x^1 = 0, x^5 = 0$ and $x^1 = \pi/\omega, x^5 = 0$. The dimensionless parameter is $mR = \frac{2M|p_-|}{\omega^2}$. We need to find the spectrum of this theory on a cylinder of circumference $R = 2\pi\alpha'|p_-|$. The excited levels of the interacting part H_2 of the light-cone gauge hamiltonian can be computed exactly by the method in [15]. While we do not have explicit results for arbitrary β, we have checked the values $\beta^2 = 8\pi$ and 16π, where the analysis is simplest. We have found that the dependence of the excited levels on mR looks similar to those presented in figure 1. As in section 3 they all decouple in the point particle limit $p_- \to 0$. By proceeding in a similar manner as in section 4 we see that there are *128* modes corresponding to the supergravity part of the superstring spectrum. This again implies that the full IIB supergravity spectrum is not obtained, as some of these modes as missing.

6. Conclusions

We analyzed the spectrum of string theories on a class of pp-wave backgrounds which result in $\mathcal{N} = (2, 2)$ supersymmetric interacting theories on the worldsheet given by the superpotential $z^3 - z$. The energy eigenvalue configuration of the light-cone gauge string hamiltonian can be determined exactly. Knowing these energies will hopefully be useful in finding the full string spectrum. An interesting limit which may be connected to strings on discrete target spaces and perhaps to non-critical string theories, is obtained when the kink mass $m \to \infty$. In this limit (or strings with very large p_-) the string, in its excited states, does not see two of the eight transverse directions. We found the supergravity modes of the string spectrum obtained by taking the $\alpha' \to 0$ and $p_- \to 0$ limit. In the former case we found the full type-IIB supergravity spectrum, as expected. However, in the case of the second limit, $p_- \to 0$, which is the UV limit on the worldsheet, we do not obtain the full IIB supergravity spectrum: as we have detailed, some of the modes are suppressed. In this case we analyzed the “massless” string modes and their space-time field interpretation under the global symmetry of the background given by the group G. The absence of some modes of IIB supergravity spectrum, obtained as $p_- \to 0$, seems also to be true when the worldsheet theory is the supersymmetric sine-Gordon model, at least in the limit when $\beta^2 \geq 8\pi$, or strong enough coupling β.

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